Name: ___Key____

INSTRUCTIONS

1. Nonprogrammable calculators (or a programmable calculator cleared in front of the professor before class) are allowed. Exam is closed book/closed notes.
2. There are 4 pages front and back for the exam in addition to the cover sheet.
3. You may leave answers in fractional form or provide three (3) decimal places.
3. Work is required to receive credit. Partial credit will be given for work that is partially correct. Points will be deducted for incorrect work even if the final answer is correct.
4. If we cannot read your work, it will be marked wrong.
5. Please attach all scrap paper to the exam.
6. Good Luck!

Page	Possible	Score
3	22	
4	32	
5	26	
6	24	
Total	104	

Note: This includes 4 points bonus. The Midterm is out of 100 points.

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(5 pts.) 1. Please write a question asking for the following sample space and event A in complete English sentences: S = {1, …, 52}, A = {x|x is even}

There are many possible answers for this part. The following are 3 simple ones.

1. Write down the sample space for a standard deck of cards. Write down the event that the card is red. This is correct because exactly half of the cards are red.

2. Write down the sample space for a rolling a 52-sided die. Write down the event that the die roll is an even number.

3. Write down the sample space for your class of size 52 after they have been numbered in your grade book. Write down the event that the student has an even number assigned to them.

(10 pts.) 2. In a study of pleas and prison sentences, it is found that 40% of the subjects studied were sent to prison. Among those sent to prison, 35% plead guilty. Among those not sent to prison, 55% plead guilty. If a person in this study was selected at random and entered a guilty plea, what is the probability that they were **NOT** sent to prison.

P: subject goes to prison G: subject is guilty

$P(P) = 0.4$ $P(P^C) = 1 – P(P) = 1 – 0.04 = 0.6$ $P(G|P) = 0.35$ $P(G|P^C) = 0.55$

$$P(P^C|G) = \frac{P(G|P^C)P(P^C)}{P(G|P^C)P(P^C) + P(G|P)P(P)} = \frac{(0.55)(0.6)}{(0.55)(0.6) + (0.35)(0.4)} = 0.702$$

(7 pts.) 3. A class consists of 35 students of which 17 are boys and 18 are girls. If you select 4 students at random for a group, what is the probability that the first 3 are girls and the last one is a boy?

$$P(G_1 \cap G_2 \cap G_3 \cap B_4) = P(G_1)P(G_2|G_1)P(G_3|G_1 \cap G_2)P(B_4|G_1 \cap G_2 \cap G_3) = \frac{18}{35} \cdot \frac{17}{34} \cdot \frac{16}{33} \cdot \frac{17}{32}$$
$$= 0.06623 = \frac{153}{2310}$$

(23 pts.) 4. The following table gives the probabilities for favorite winter sport for 401 middle school students:

Grade	Sledding	Cross Country Skiing	Ice Skating	Reading	TOTAL
6^{th}	0.13	0.06	0.08	0.04	0.31
7^{th}	0.12	0.07	0.11	0.07	0.37
8^{th}	0.07	0.10	0.09	0.06	0.32
TOTAL	0.32	0.23	0.28	0.17	1.00

a) What type of probability is this? Please briefly explain your answer.

empirical because the probabilities are based on the data.

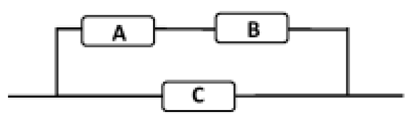
b) Assuming that the student is a 6^{th} grader, what is the probability that the student does **NOT** prefer sledding?

$$P(S^c|6) = 1 - P(S|6) = 1 - \frac{P(S \cap 6)}{P(6)} = 1 - \frac{0.13}{0.31} = \frac{18}{31} = 0.581$$

c) Assuming that the student is **NOT** a 6^{th} grader, what is the probability that the student prefers sledding?

$$P(S|6^C) = \frac{P(S \cap 6^C)}{P(6^C)} = \frac{P(S \cap 7) + P(S \cap 8)}{P(7) + P(8)} = \frac{0.12 + 0.07}{0.37 + 0.32} = \frac{19}{69} = 0.275$$

(9 pts.) 5. In the following circuit, all of the components are independent of each other. The probability that each of the components fails is 0.1. What is the probability that the circuit will succeed?



P(Success) = 1 – P(fail) = 1 – 0.1 = 0.9

$P((A \cap B) \cup C) = P(A \cap B) + P(C) - P(A \cap B \cap C)$

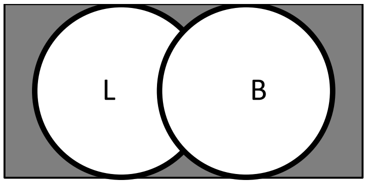
$= P(A)P(B) + P(C) - P(A)P(B)P(C)$

$= 0.9^2 + 0.9 - 0.9^3$

$=0.981$

(26 pts.) 6. In a kindergarten class, the children like two types of candy, lollipops and bubble gum. In this class, 36% of the children like bubble gum, 12% like both of the types of candy and 23% of the children don't like either of these types of candy. A child is chosen at random.

L: Lollipop   B: bubble gum
$P(B) = 0.36$ $P(L \cap B) = 0.12$



$P(L^C \cap B^c) = 0.23$

a) What is the probability that the child likes at least one of the types of candy? Hint: Shade in what the 23% represents on a Venn diagram.

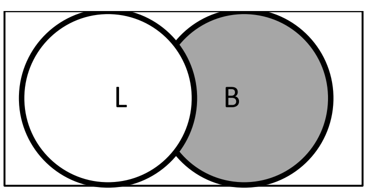
$P(L \cup B) = 1 – P(L^C \cap B^C) = 1 – 0.23 = 0.77$

b) What is the probability that the child likes lollipops?

$P(L \cup B) = P(L) + P(B) – P(L \cap B)$

$P(L) = P(L \cup B) + P(L \cap B) – P(B)$
$= 0.77 + 0.12 – 0.77$
$= 0.53$

c) What is the probability that a child likes bubble gum but does NOT like lollipops?



$P(B \cap L^C) = P(B) – P(L \cap B) = 0.36 – 0.12 = 0.24$
OR
$P(B \cap L^C) – P(L \cup B) – P(L) = 0.77 – 053 = 0.24$

d) Is the fact that a child likes lollipops independent of the fact that a child likes bubble gum? There will be no credit unless a correct explanation is provided.

Two events are independent if $P(B)\,P(L) = P(B \cap L)$

$P(B)\,P(L) = (0.36)(0.53) = 0.1908$
Since $P(B \cap L) = 0.12$, they are not equal so B and L are NOT independent.

(7 pts.) 7. Prove the following statement: $P(A^c) = 1 - P(A)$. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed. Hint: Rearrange the equation.

Step	Reason
$1 = P(S)$	completeness axiom
$= P(A \cup A^C)$	definition of complement
$= P(A) + P(A^C)$	definition of complement: A and A^C are disjoint additivity axiom
$P(A^C) = 1 - P(A)$	algebraic rearrangement

(10 pts.) 8. A meteorologist records whether it snows or not on 4 successive days in winter.

a) Using set theory notation, what is the sample space for this situation?

$S = \{x_1, x_2, x_3, x_4 \mid x_i \in \{\text{snow, no snow}\}\}$

b) Using set theory notation, describe the event that it does snow for the first two days.

$A = \{\text{snow, snow}, x_3, x_4 \mid x_i \in \{\text{snow, no snow}\}\}$

(7 pts.) 9. If two events, A and B, are disjoint, is it possible for them to also be independent if $P(A) \neq 0$ and $P(B) \neq 0$? Use mathematical reasoning to explain your answer, that is, you cannot provide an example or counter-example to explain your choice but need to use mathematical equations.

If A and B are disjoint $\quad P(A \cap B) = 0$
If A and B are independent $\quad P(A)\,P(B) = P(A \cap B)$

Therefore, if A and B are disjoint, they can only be independent if at least one of $P(A) = 0$ or $P(B) = 0$.

However, the question states that $P(A) \neq 0$ and $P(B) \neq 0$, therefore if A and B are disjoint, they cannot be independent.